

ALS Beamline Note

Angle calibration of the long trace profiler  
using a diffraction grating

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20 August, 1992

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## 1. Introduction

The long trace profiler (LTP) is an instrument for measuring the surface profile (either slope or height) of a long optic. The LTP measures slope of the surface at several points on the surface under test (SUT), so the height profile is obtained by integrating the slope (as a function of position) to get the height function. Thus it is critical that the LTP accurately measure the slope at each point on the SUT if the profiles are to be correct.

The LTP measures slope by sending two parallel, collimated, and temporally coherent beams of light toward the SUT. The beams are reflected at the SUT, and the slope of the SUT at that point will determine the angle of the reflected beams going back into the LTP. The angle of the beams going back into the LTP is determined by the relative phase between the two beams. This angle of optical deviation will be twice the angle of the SUT slope.

One way to assure that the slope is accurately measured is to calibrate the LTP by sending the light beams back into the LTP at a precisely known angle. This may be done with a diffraction grating whose characteristics are known precisely. The beams coming out of the LTP are incident on the grating, and the beams diffracted from the grating go back into the LTP with precisely known angles.

## 2. Theory

The grating equation

$$m \lambda f_g = \sin \alpha + \sin \beta \quad (1)$$

is the only equation we need, and gives a precise relationship between all the parameters we are concerned with. Here,  $m$  is the diffraction order,  $\lambda$  is the wavelength of the light beams,  $f_g$  is the grating groove frequency,  $\alpha$  is the angle of the light beams incident on the grating with respect to the grating surface normal, and  $\beta$  is the diffracted beams' angle with respect to the grating surface normal.

Since the acceptance angle of the LTP is limited to  $\pm 10$  mrad (optical deviation), the grating groove frequency must be somewhat low. In order to get three diffracted orders (e.g.,  $m=-1$ ,  $m=0$ ,  $m=+1$ ) into the LTP, the groove frequency should be no more than 14.9 gr/mm. The closest grating to this I could find was in stock at Milton Roy Company (master fabricated at Fresnel

Products Corp.); its groove frequency is 11.58 gr/mm, blazed at 1.333 degrees. This small frequency gives rise to many diffracted orders that are visible in a short angular spread.

If the grating is used in the Littrow condition ( $\alpha = \beta$ ), then there is one less parameter in the analysis. In this case

$$m \lambda f_g = 2 \sin \alpha . \quad (2)$$

A trivial situation would be to set up the grating in Littrow for  $m=0$ . Then the diffracted orders going into the LTP would be  $m=-1$ ,  $m=0$ , and  $m=+1$ . An advantage here is that symmetry allows the diffracted beams  $m=-1$ ,  $m=+1$  to be at the same angle with respect to  $m=0$ . But when one gets a relatively cheap grating from stock, one can't be too choosy. If one is lucky enough to get a grating with about the right groove frequency, then one would be very lucky to get just the right groove shape. So this analysis will assume that any groove frequency and any blaze may be used.

When the grating is inserted in place of the SUT (or the REF mirror), the grating tilt is adjusted so that the brightest order is going right down the center of the LTP optical system. If  $\gamma$  is the blaze angle, then  $\alpha = \beta = \gamma$  in Littrow, but only approximately. See figure 1. Actually, the brightest order is

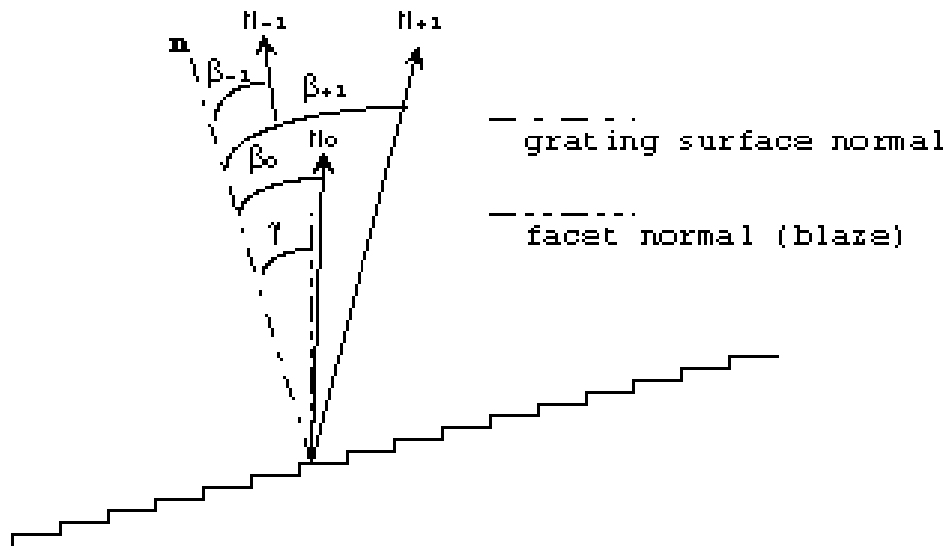


Figure 1. Geometry of the grating surface and parameters in the grating equation. Three diffracted orders are shown;  $\alpha = \beta_0$ .

calculated by finding which order is closest to both the blaze condition and the Littrow condition:

$$m_f = \frac{2 \sin \gamma}{\lambda f_g}. \quad (3)$$

$m_f$  will most likely be fractional, hence the subscript. We then round this number to the nearest whole order. For example, if  $m_f$  is 3.51, then we would realize  $M_0=4$  for the brightest order; but in this case the 3rd order would be almost as bright.

Now two important things are known: 1) which order  $M_0$  is the brightest, and 2) the fact that  $\alpha = \beta$  for the brightest order. For the central order,  $\alpha = \beta$  (exactly) and

$$\sin \alpha = \frac{M_0 \lambda f_g}{2}. \quad (4)$$

Then Equation (1) may be used to find the difference in angle between the central order and either of the other two orders.

$$M_n \lambda f_g = \sin \alpha + \sin \beta = \frac{M_0 \lambda f_g}{2} + \sin \beta_n, \quad (5)$$

where  $M_n$  is another order relative to  $M_0$ , and  $\beta_n$  is the diffracted angle for  $M_n$ . Solving for  $\beta_n$  gives

$$\beta_n = \arcsin[\lambda f_g (M_n - M_0 / 2)]; \quad (6)$$

and then

$$\beta_n - \beta_0 = \arcsin[\lambda f_g (M_n - M_0 / 2)] - \arcsin[\lambda f_g M_0 / 2]. \quad (7)$$

Of course,  $\beta_0$  is the angle for the central order. Equation (7) gives the angle between the central order and the order to the left ( $n=-1$ ) or the order to the right ( $n=+1$ ). In order to preserve precision, do not make approximations for  $\arcsin$ .

### 3. Our example

First we find  $M_0$  from Equation (3).  $\gamma = 1.333$  degrees,  $f_g = 11.58$  gr/mm, and  $\lambda = 670$  nm (wavelength of the laser in the LTP).

$$mf = \frac{2 \sin \gamma}{\lambda f_g} = 5.9982, \quad (8)$$

and so  $M_0 = 6$ .

The central order angle will be

$$\beta_0 = \arcsin[\lambda f_g M_0 / 2] = 0.232779 = 23.2779 \text{ mrad}. \quad (9)$$

Then the order to the left will be

$$\beta_{-1} = \arcsin[\lambda f_g (M_{-1} - M_0 / 2)] = 15.5178 \text{ mrad}, \quad (10)$$

and the order to the right will be

$$\beta_{+1} = \arcsin[\lambda f_g (M_{+1} - M_0 / 2)] = 31.0394 \text{ mrad}. \quad (11)$$

These angles relative to the central order will be

$$\beta_{-1} - \beta_0 = -7.7601 \text{ mrad}, \text{ and} \quad (12)$$

$$\beta_{+1} - \beta_0 = 7.7615 \text{ mrad}. \quad (13)$$

### 4. Error

The accuracy of the results in Equations (12) and (13) will be in the order of accuracy of  $b_n$  as calculated in Equation (6). The total differential of Equation (6) is

$$d\{\sin \beta_n\} = d\{\lambda f_g (M_n - M_0 / 2)\}, \text{ or} \quad (14)$$

$$d\{\beta_n\} \cos \beta_n = (M_n - M_0 / 2) (\lambda d\{f_g\} + f_g d\{\lambda\}). \quad (15)$$

Therefore,

$$d\{\beta_n\} \approx (M_n - M_0/2) (\lambda d\{f_g\} + f_g d\{\lambda\}). \quad (16)$$

Values for the uncertainties of the grating  $d\{f_g\}$  and the wavelength  $d\{\lambda\}$  are given by the respective manufacturers. The wavelength of the laser diode is within  $\pm 2$  nm. The grating frequency tolerance, however, is not as well defined. My experience with the production and quality assurance of these gratings is that the least significant digit they quote may be taken as true. Therefore,  $d\{f_g\} = .01$  gr/mm. Putting these values into Equation (16) gives

$$\begin{aligned} d\{\beta_0\} &= (6 - 3) (.00067 * .01 + .000002 * 11.58) = .090 \text{ mrad}; \\ d\{\beta_{-1}\} &= (5 - 3) (.00067 * .01 + .000002 * 11.58) = .060 \text{ mrad}; \\ d\{\beta_{+1}\} &= (7 - 3) (.00067 * .01 + .000002 * 11.58) = .119 \text{ mrad}. \end{aligned} \quad (17)$$

This angular uncertainty overwhelms the slight skew between orders seen in Equations (12) and (13). Another source of error is how much misalignment there is in setting  $\alpha = \beta_0$ . With careful alignment, this error can be in the order of the uncertainty of the LTP's optical axis, which is less than the above uncertainties (Equation (17)).

## 5. Conclusion

Using a diffraction grating for calibrating angle is generally a good idea. However, uncertainties in the system limit the accuracy of this method. As seen from Equation (16), using the low orders of the grating will give the least uncertainty,  $d\{\beta_0\} = .030$  mrad. If the LTP laser has plenty of intensity, the low orders may be used with the grating in our example. This method will provide a quick way of calibrating the LTP so that slopes may be measured with an accuracy of better than 0.5%. Much better accuracy than this will require a well characterized grating and a stabilized, single frequency laser.